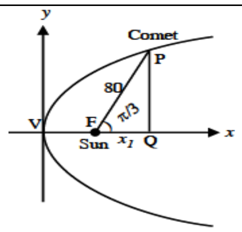
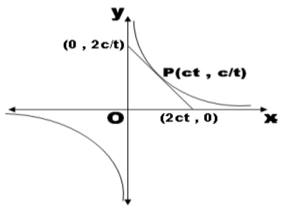
PUBLIC EXAMINATION 2019

XII - MATHEMATICS **TENTATIVE ANSWER KEY** **07.03.2019**

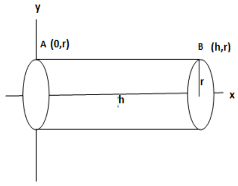
ANSWER KEY

Q.No	PART – I (TYPE - A)	MARKS
1.	(4) $\frac{1}{11}$	1
2.	(1) $\frac{9}{4}$	1
3.	(4) 3	1
4.	(3) g(x) is an identity function	1
5.	(4) second quadrant	1
6.	(1) pv(~p)	1
7.	(2) 8	1
8.	(2) xz - plane	1
9.	(1) The curve has a point of inflection in which y'' does not exit	1
10.	(2) collinear	1
11.	(4) trivial solution and infinitely many non-trivial solutions	1
12.	(3) $(y')^2 - xy' + y = 0$	1
13.	(3) 2,2	1
14.	(4) 0	1
15.	(1) 3	1
16.	(4) 16π	1
17.	(2) $\frac{\pi}{2}$	1
18.	(4) x+ 3 =0	1
19.	(1) $\frac{1}{k}I$	1
20.	(3) 1	1
	PART – II	
21.	x+y+z = 30 x+2y+5z=100	1 1
22.	$\vec{a} = \lambda\vec{b}$ $3\vec{i} + 2\vec{j} + 9\vec{k} = \lambda\left(\vec{i} + m\vec{j} + 3\vec{k}\right)$ $\lambda = 3$ $m = \frac{2}{3}$	 1 1

23.	$\frac{1+i}{1-i}=i$ $\left(\frac{1+i}{1-i}\right)^n = i^4 = 1$	1 1
24.		2
25.	$f(x)=\sin x$ $f'(x)=\cos x$ $f'(x)=0 \Rightarrow x=(2n+1)\frac{\pi}{2}, n\in z$	1 1
26.	Domain: $(-\infty,\infty)$ Extent: Horizontal Extent: $-\infty < x < \infty$ Vertical Extent: $-\infty < y < \infty$	1 1
27.	$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\cot x}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\cot\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}$ $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$	1 1
28.	$G = Q-\{0\}$ Let $a\in G$, $-a\in G$ $a+(-a)=0\notin G$ $\therefore$ closure axiom is not true.	1 1
29.	$F(3)=p(x\le 3)=\int_0^3 3e^{-3x}dx$ $=3\left[\frac{e^{-3x}}{-3}\right]_0^3$ $=1-e^{-9}$	1 1
30.	$f(x)=\lvert x-2\rvert+\lvert x-5\rvert \, in [1,6]$ f(x) is continuos on [1,6] f(x) is not differentiable on (1,6) $\therefore$Rolles theorem not satisfied	1 1
	PART – III	
31.	Let $A=\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix}\Rightarrow \rho(A)=2$, $B=\begin{bmatrix}2&3&4\\5&6&7\\8&9&10\end{bmatrix}\Rightarrow \rho(B)=2$	1 1

	$A+B=\begin{bmatrix}3&5&7\\9&11&13\\15&17&19\end{bmatrix}\Rightarrow \rho(A+B)=2$ $\rho(A)+\rho(B)=4$ $\rho(A+B)\neq\rho(A)+\rho(B)$	1
32.	$\vec{a}\times\vec{b}=-\vec{i}+2\vec{j}+2\vec{k}, \lvert\vec{a}\times\vec{b}\rvert=3, \mu=6$ Required vector = $=\pm\mu\left(\dfrac{\vec{a}\times\vec{b}}{\lvert\vec{a}\times\vec{b}\rvert}\right)$ $=\pm(-2\vec{i}+4\vec{j}+4\vec{k})$	1 2
33.	$z=\sin\theta-i\cos\theta, \dfrac{1}{z}=\sin\theta+i\cos\theta$ $\left(\dfrac{1+\sin\theta-i\cos\theta}{1+\sin\theta+i\cos\theta}\right)^n=(z)^n=(\sin\theta-i\cos\theta)^n$ $=\left[\cos\left(\dfrac{\pi}{2}-\theta\right)-i\sin\left(\dfrac{\pi}{2}-\theta\right)\right]^n$ $=\cos n\left(\dfrac{\pi}{2}-\theta\right)-i\sin n\left(\dfrac{\pi}{2}-\theta\right)$	1 1 1
34.	The equation of tangent at $P\left(ct,\frac{c}{t}\right)$ is $x+yt^2=2ct$ The co-ordinates A $(2ct, 0)$, B $\left(0,\frac{2c}{t}\right)$  The mid-point of AB = $\left(ct,\dfrac{c}{t}\right)$ which is the point P. This shows that the tangent is bisected at the point of contact.	1 1 1
35.	$f(x)=\tan^{-1}(\sin x+\cos x), x>0$ $f'(x)=\dfrac{\cos x-\sin x}{1+(\sin x+\cos x)^2}$ $=\dfrac{\cos x-\sin x}{2+\sin 2x}>0$ f(x) is strictly increasing function in $\left(0,\dfrac{\pi}{4}\right)$	2 1
36.	$f(x,y)=\dfrac{1}{\sqrt{x^2+y^2}}$ $f(tx,ty)=t^{-1}f(x,y)$ f is a homogenous function of degree n = -1 By euler's thm, $x\dfrac{\partial f}{\partial x}+y\dfrac{\partial f}{\partial y}=-f$	1 2
37.	The equation of AB is $\dfrac{y-r}{r-r}=\dfrac{x-0}{h-0}$ $y=r$	1

Volume of cylinder = $v = \pi\int_0^h y^2 dx = \pi\int_0^h r^2 dx$ — 1



$= \pi r^2 [x]_0^h$

$= \pi r^2 h$ — 1

38.

p	q	p ∧q	p ∨q	(p∧q) →(p ∨q)
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

— 2

The last column contains only T.

∴ (p∧q) →(p ∨q) is a tautology. — 1

39.

n = 120, $p = \frac{2}{6} = \frac{1}{3}, q = \frac{4}{6} = \frac{2}{3}$ — 1

mean = np = 40 — 1

variance = npq = $\frac{80}{3}$ — 1

40.

$yx^3 dx = -e^{-x} dy$

$x^3 e^x dx = -\frac{dy}{y}$ — 1

$\int x^3 e^x dx = -\int \frac{dy}{y}$

$e^x\left[x^3 - 3x^2 + 6x - 6\right] + \log y = c$ — 2

PART – IV

41.

(a) Augmented matrix is $[A,B] = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 4 & 3 & \mu & 0 \\ 2 & 1 & 2 & 0 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & \mu-12 & 0 \\ 0 & -1 & -4 & 0 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 4R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$

$\sim \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & \mu-12 & 0 \\ 0 & 0 & 8-\mu & 0 \end{bmatrix} R_3 \to R_3 - R_2$ — 2

Case(i): Let $\mu \neq 8$

$\rho[A, B] = 3$ and $\rho(A) = 3$,

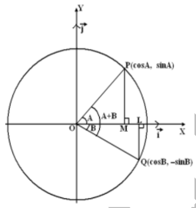
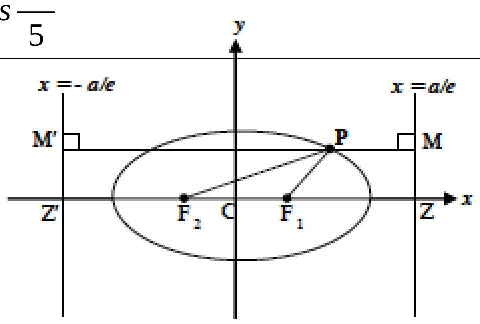
The given system is consistent and has trivial solution. — 1

i.e., $x = 0, y = 0, z = 0$

Case(ii): Let $\mu = 8$

$\rho[A, B] = 2, \quad \rho(A) = 2 < 3 (= \text{number of unknowns})$

Corresponding equations are $x + y + 3z = 0; \quad y + 4z = 0;$

	Taking $z = k,\ \ k \in R - \{0\}$ $\Rightarrow\ \ y = -4k \quad and \quad x = k$ The solutions set is $(x,\ y,\ z) = (k,\ \ -4k,\ \ k),\ \ k \in R - \{0\}$ which are non-trivial. **The given system is consistent and has infinitely many non-trivial solutions.**	1 1
	(b) Diagram  $\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP} = cosA\vec{i} + sinA\vec{j}$ $\overrightarrow{OQ} = \overrightarrow{OL} + \overrightarrow{LQ} = cosB\vec{i} - sinB\vec{j}$ $\overrightarrow{OQ} \times \overrightarrow{OP} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ cosB & -sinB & 0 \\ cosA & sinA & 0 \end{vmatrix} = \vec{k}[\sin AcosB + cosAsinB] ---\rightarrow (1)$ By definition, $\overrightarrow{OQ} \times \overrightarrow{OP} = \lvert\overrightarrow{OQ}\rvert\lvert\overrightarrow{OP}\rvert \sin(A+B)\vec{k} = \sin(A+B)\vec{k} ---\rightarrow (2)$ From (1) and (2), **sin(A+B)=** $\mathbf{\sin AcosB + cosAsinB}$	1 1 1 1 1
42.	(a) $(x_1, y_1, z_1) = (-1, 1, -1), (x_2, y_2, z_2) = (2, 2, 1)$ and $(l_1, m_1, n_1) = (2, 3, -2)$ The equation of the plane is $\begin{vmatrix} x+1 & y-1 & z+1 \\ 3 & 1 & 2 \\ 2 & 3 & -2 \end{vmatrix} = 0$ $\mathbf{8x - 10y - 7z + 11 = 0}$	3 2
	(b) $x^6(x^5-1)+(x^5-1)$ $(x^6+1)(x^5-1)=0$ $x = (-1)^{\frac{1}{6}} = (cis\pi)^{\frac{1}{6}}$ $= (cis(2k\pi+\pi))^{\frac{1}{6}}\ k = 0,1,2,3,4,5,$ $x = cis\frac{\pi}{6}, cis\frac{3\pi}{6}, cis\frac{5\pi}{6}, cis\frac{7\pi}{6}, cis\frac{7\pi}{6}, cis\frac{9\pi}{6}, cis\frac{11\pi}{6}$ $x = (cis0)^{\frac{1}{5}} = (cis(2k\pi))^{\frac{1}{5}}$ $= cis\frac{2k\pi}{5}, k = 0,1,2,3,4$ $x = cis0, cis\frac{2\pi}{5}, cis\frac{4\pi}{5}, cis\frac{6\pi}{5}, cis\frac{8\pi}{5}$	1 1 1 1 1
43.	(a) Diagram  $F_1P + F_2P = 2a$ $x = \frac{a}{e}, x = -\frac{a}{e}$ $\frac{F_1P}{PM} = e, \frac{F_2P}{PM'} = e$ $F_1P = ePM, F_2P = ePM'$ $F_1P + F_2P = e(PM + PM')$	1 1

	$= e(MM')$ $= e.\frac{2a}{e}$ =length of the major axis $2a = 9, a = \frac{9}{2}, ae = 3$ $b^2 = a^2 - (ae)^2 = \frac{81}{4} - 9$ $b^2 = \frac{45}{4}$ The equation is $\frac{x^2}{\left(\frac{81}{4}\right)} + \frac{y^2}{\left(\frac{45}{4}\right)} = 1$	 1 1 1
	(b) Diagram Area of the rectangle $A = 4r^2 sin\theta cos\theta$ $A(\theta) = 2r^2 sin2\theta$ $A'(\theta) = 4r^2 cos2\theta$ $A''(\theta) = -8r^2 sin2\theta$ $A'(\theta) = 0 \Rightarrow 4r^2 cos2\theta = 0 \Rightarrow 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$ When $\theta = \frac{\pi}{4}, A''\left(\frac{\pi}{4}\right) = -8r^2 < 0$ When $\theta = \frac{\pi}{4}$, A is maximum. When $\theta = \frac{\pi}{4}$, $Length = 2x = \sqrt{2}r;$ $Breadth = 2y = \sqrt{2}r$ $Area\ A = 2r^2$	1 1 1 1 1
44.	(a) $x = 100t - \frac{25}{2}t^2$ (i) Velocity $v = \frac{dx}{dt} = 100 - 25t\ m/s$ Initial velocity $= \left(\frac{dx}{dt}\right)_{t=0} = 100m/s$ (ii) At maximum height, $\frac{dx}{dt} = 0 \Rightarrow 100 - 25t = 0 \Rightarrow t = 4\ s$ (iii) Maximum height reached $= x(4) = 400 - 200 = 200\ m$ (iv) When the missile reaches the ground, $x = 0$. $100t - \frac{25}{2}t^2 = 0 \Rightarrow t = 0$ or $t = 8$ $t = 0$ is not admissible. $\therefore t = 8$ sec. Velocity of the missile when it reaches the ground $= \left(\frac{dx}{dt}\right)_{t=8} = 100 - 25(8) = -100m/s$.	 1 1 1 1 1

(b) $\frac{(y+1)^2}{16}-\frac{(x-1)^2}{9}=1$ 1

$a^2=16;\ b^2=9 \Rightarrow e=\sqrt{\frac{a^2+b^2}{a^2}}=\sqrt{\frac{25}{16}}=\frac{5}{4}\Rightarrow ae=5$ 1

	Referred to X,Y axis	Referred to x,y axis $x=X+1, y=Y$
Centre	C(0,0)	C(1,-1)
Foci	$(0,\pm ae)=(0,\pm 5$	$F_1(1,4)$ $F_2(1,-6)$
Vertices	$(0,\pm a)=(0,\pm 4)$	$(1,3),(1,-5)$
Eccentricity	$e=\frac{5}{4}$	$e=\frac{5}{4}$

2

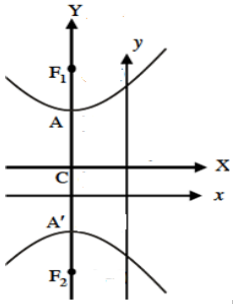
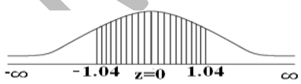


Diagram 1

45. (a) Given $\mu=34$, $\sigma=16$ & $N=1000$

Value of z_1 from the area table for the area 0.35 is -1.04 ,Similarly, $z_2=1.04$ 2



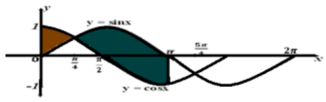
$z_1=\frac{X_1-34}{16}=-1.04$ $\Rightarrow$

$X_1-34=-16.64 \Rightarrow X_1=17.36$ 1

$z_2=\frac{X_2-34}{16}=\ 1.04 \Rightarrow X_2-34=\ 16.64 \Rightarrow X_2=50.64$ 1

∴ Central 70% of the candidates scored between 17.36 & 50.64. 1

(b) Diagram 1



$sinx=cosx=\frac{1}{\sqrt{2}} \Rightarrow x=\frac{\pi}{4}$. 1

Requiredarea= $\int_0^{\frac{\pi}{4}}(cosx-sinx)dx+\int_{\frac{\pi}{4}}^{\pi}(sinx-cosx)dx$ 1

$=[sinx+cosx]_0^{\pi/4}+[-cosx-sinx]_{\pi/4}^{\pi}$ 1

$=\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)-(0+1)+(1-0)-\left(-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)=\frac{2}{\sqrt{2}}+\frac{2}{\sqrt{2}}$

$=\ 2\sqrt{2}$ sq.units 1

46. (a) $w=x+2y+z^2$

$\frac{\partial w}{\partial x}=1;\ \frac{\partial w}{\partial y}=2;\frac{\partial w}{\partial z}=2z$ 2

$\frac{dx}{dt}=-sint;\ \frac{dy}{dt}=cost\ ;\frac{dz}{dt}=1$

$\frac{dw}{dt}=\frac{\partial w}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial w}{\partial y}\cdot\frac{dy}{dt}+\frac{\partial w}{\partial z}\cdot\frac{dz}{dt}$

$\frac{dw}{dt}$ =-sint+2cost+2z 1

= -sint+2cost+2t

<u>Verification</u>

$w=\cos t+2\sin t+t^2$

$\frac{dw}{dt}$ = -sint+2cost+2t 2

	(b) Let T be the temperature of the coffee at any time t. $\frac{dT}{dt} \propto (T-15)$ since S = 15°C ⇒ $\frac{dT}{dt} = k(T-15) \Rightarrow T-15 = ce^{kt}$ When $t = 0,\ T = 100 \Rightarrow 100-15 = ce^{0} \Rightarrow c = 85$ $\therefore T-15 = 85e^{kt}$ When t = 5, T = 60 ⇒ $60-15 = 85e^{5k} \Rightarrow 45 = 85e^{5k}$ $\Rightarrow e^{5k} = \frac{45}{85}$ When t = 10, $T - 15 = 85e^{10k}$ $T = 15 + 85\left(e^{5k}\right)^2$ $T = 15 + 85\left(\frac{45}{85}\right)^2$ $= 15 + 23.82°C = 38.82°C$	2 1 1 1
47.	(a) 1. The identity element of the group is unique . 2. The inverse element of an element of a group is unique. 3. In a group G, $(a^{-1})^{-1} = a$ for every $a \in G$ 4. Let G be a group. Then for all $a,b,c \in G$ (i) $a*b = a*c \Rightarrow b = c$ (ii) $b*a = c*a \Rightarrow b = c$ 5. Let G be a group $a,b \in G$. Then $(a*b)^{-1} = b^{-1}*a^{-1}$	1 1 1 1 1
	(b) The characteristic equation is $5p^2 - 8p - 4 = 0$ $(5p+2)(p-2) = 0$ $p = -\frac{2}{5},\ p = 2$ $C.F = Ae^{2x} + Be^{-\frac{2}{5}x}$ $PI_1 = \frac{5e^{-\frac{2}{5}x}}{10D-8} = \frac{-5}{12}xe^{-\frac{2}{5}x}$ $PI_2 = \frac{2e^{x}}{5D^2-8D-4} = -\frac{2}{7}e^{x}$ $PI_3 = \frac{3}{5D^2-8D-4} = -\frac{3}{4}$ $y = Ae^{2x} + Be^{-\frac{2}{5}x} - \frac{5}{12}xe^{-\frac{2}{5}x} - \frac{2}{7}e^{x} - \frac{3}{4}$	1 1 1 1 1

t	T	S
0	100	15
5	60	15
10	?	15

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